The Role of Magnetosonic Shocks in the dynamics and stability of staged Z-pinch

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Overview

- Role and Formation of the Shocks in staged Z-pinches
- Propagation of nonlinear Magnetoacoustic waves is described by a KDV-Burgers Equation
- Stable compression and heating of the target plasma with high-Z liner (Ar, Kr or Xe)
- Axial Magnetic field provides enhanced stability and Thermal insulation
- Pre-heating and compression by shocks
- Formation of high-energy-density, stable plasma
- Charged fusion products trap due to large magnetic field that may lead to Ignition

STAGED Z-PINCH

Schlieren Image of the Stable Target Implosion (UCI Experiment)
Kr Liner imploding on Deuterium target

Shock stabilized secondary piston

Unstable outer liner

LOW-Z TARGET

HIGH-Z PLASMA LINER

DIFFUSED FIELDS

AZIMUTHAL \( (B_\theta) \) FIELD

AXIAL \( (B_z) \) FIELD

AXIAL CURRENT

Stable Target

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Stable Target
Derivation of KDV-Burgers Equation from MHD equation used in MACH2

• Goal: Obtain single equation from MACH2 MHD equations that describes magnetosonic shock formation

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0 \\
\rho \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} &= -\nabla P + \mathbf{J} \times \mathbf{B} \\
\rho \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \frac{T}{m_i} &= -P(\nabla \cdot \mathbf{v}) + \eta J^2 + \nabla \cdot (\kappa \nabla T) \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J} \\
\mathbf{E} + \mathbf{v} \times \mathbf{B} &= \eta \mathbf{J}
\end{align*}
\]

• Take
  • \(\mathbf{v}, \rho, T = v(x)\hat{x}, \rho(x), T(x)\)
  • \(\mathbf{B} = B(x)\hat{y}\)
  • \(\mathbf{J} = J(x)\hat{z}\)

• Replace
  • \(P \equiv c_s^2 \rho\)
Derivation of KDV-Burgers Equation

• Non-dimensionalize these equations
  • \( \rho = \rho' \rho_0, v = u v_A, B = B' B_0, T = T' T_0 \)
  • \( t = \frac{t'L}{v_A}, x = x'L \)

• Introduce stretching coordinate
  • \( \xi = -\epsilon^\alpha (x + v_0 t), \tau = \epsilon^{\alpha + 1} t' \)

\[ \frac{\partial}{\partial t'} \rightarrow -\epsilon^\alpha v_0 \frac{\partial}{\partial \xi} + \epsilon^{\alpha + 1} \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x'} \rightarrow -\epsilon^\alpha \frac{\partial}{\partial \xi} \]

• Expand the dependent variables in a power series
  • \( \rho' = 1 + \epsilon \rho_1' + \epsilon^2 \rho_2' + \cdots \), same for \( B', T' \)
  • \( u = +\epsilon u_1 + \epsilon^2 u_2 + \cdots \)

• Solve equations to first and second order of \( \epsilon \)

  • To first order, obtain convenient solution:
    \[ \rho_1' = B_1' = T_1' = -\frac{u_1}{v_0} = -\frac{1}{\sqrt{\beta + 1}} u_1, (\beta \equiv \frac{n k T}{P_B}) \]

  • With some algebra and using first order solution, second order equations reduce to
    \[ \frac{\partial \rho_1'}{\partial \tau} + \frac{3}{2} v_0 \rho_1' \frac{\partial \rho_1'}{\partial \xi} - \hat{\kappa} + \hat{\eta} \frac{\partial^2 \rho_1'}{\partial \xi^2} = 0 \]
Derivation of KDV-Burgers Equation

\[ \frac{\partial \rho'_1}{\partial \tau} + \frac{3}{2} v_0 \rho'_1 \frac{\partial \rho'_1}{\partial \xi} - \frac{\hat{\kappa} + \hat{\eta}}{2 v_0^2} \frac{\partial^2 \rho'_1}{\partial \xi^2} = 0 \]

KDV-Burgers Equation

\[ \frac{3}{2} v_0 \text{ - strength of nonlinearity} \]

\[ \frac{\hat{\kappa} + \hat{\eta}}{2 v_0^2} \text{ - controls rate of dissipation} \]

\[ \hat{\kappa} \equiv \frac{\kappa T_0}{\rho_0 v_A^3 L}, \hat{\eta} \equiv \frac{\eta}{\mu_0 v_A L} \]

Shock development governed by several variables

- Plasma Beta \( (v_0 \equiv \sqrt{\beta + 1}) \), thermal conductivity \( (\kappa) \), electrical resistivity \( (\eta) \), Alfvén speed \( (v_A) \)

- How does choice of liner affect these parameters?
Shock wave solutions from KDV-Burgers Equation
Numerical Simulation using MACH2

- 2&1/2 dimensional, time-dependent, single fluid, MHD simulation code.
- Used in Eulerian mode.
- External capacitor bank circuit is modeled.
- Tabular (SESAME) equations of state.
- Implicit MHD with components of $B$ and $U$.
- Multi-species plasma.
- Flux-limited, single group, implicit radiation diffusion.
- The code has been benchmarked against several experiments.
MACH2 Simulation of implosion Dynamics

- **Ar/DD**
- **Kr/DD**
- **Xe/DD**

The plots show the evolution of pressure ($P_t$) over time ($Z$) for different implosion scenarios using MACH2 simulation.
Plasma Density Iso-contours at Peak Implosion time superimposed with magnetic field profile